Statistics

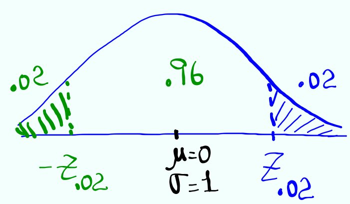
Lecture 13

Feb 19-8:47 AM

Find $Z_{\alpha/2}$ for $\alpha = .04$ $\qquad 1 - \alpha = .96$

$.04/2 = .02$

$Z_{.02}$



$Z_{.02} = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$

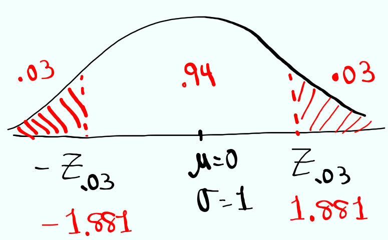
Find $\pm Z_{\alpha/2}$ for $\underbrace{\text{94\% Conf. level.}}_{\text{Middle Area}}$

$.94$

$1 - .94 = .06$

$\alpha = .06$

$\alpha/2 = .03$

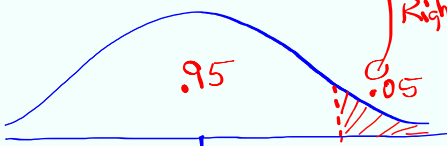


$Z_{.03} = \text{invNorm}(.97, 0, 1) = \boxed{1.881}$

Jul 22-4:34 PM

Find $t_{.05}$ with $df = 11$

Right Tail Area



σ unknown
$df = 11$

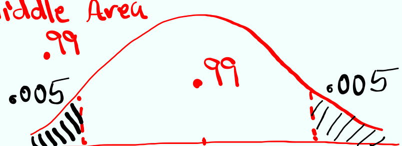
$t_{.05} = \text{invT}(.95, 11)$
$= \boxed{1.796}$

Find $\pm t_{\alpha/2}$ for $\underbrace{99\%\ \text{C-level}}_{\text{Middle Area}}$ with $df = 14$.

.99

$1 - .99 = .01$

$.01 \div 2 = .005$



$-t_{.005}$ $\quad \mu = 0 \quad$ $t_{.005}$

-2.977 $\quad$ σ unknown $\quad$ 2.977

$df = 14$

$t_{.005} = \text{invT}(.995, 14) = \boxed{2.977}$

Jul 22-4:40 PM

CNN has randomly selected 450 voters, and 40% of them said they are likely to vote for Ms. Harris for president.

$n = 450$
$\hat{p} = .4$ $\Rightarrow x = n\hat{p} = 450(.4) = 180$

Find 99% Conf. interval for the prop. of all voters that are likely to vote for Ms. Harris. C-level: .99

$\boxed{.34 < p < .46}$

1-Prop Z Int
$x = 180$
$n = 450$
C-level: .99

$E = \frac{.46 - .34}{2} = \boxed{.06}$

$\hat{p} = \frac{.46 + .34}{2} = \boxed{.4}$

Jul 22-4:47 PM

The college randomly selected 125 students and 6.5% of them were left-handed.

$n = 125$

$\hat{p} = .065$ $\Rightarrow x = n\hat{p} = 125(.065) = 8.125 = \boxed{9}$

Decimal $\Rightarrow$ Round up.

Find conf. interval for the prop. of all students that are left handed.

$\Rightarrow$ No C-level $\Rightarrow$ use .95

1 – Prop Z Int

$x = 9$

$n = 125$

.95

$\boxed{2.7\% < p < 11.7\%}$

$\boxed{.027 < p < .117}$

$\hat{p} = \frac{.117 + .027}{2} = \boxed{.072}$

$E = \frac{.117 - .027}{2} = \boxed{.045}$

Jul 22-4:54 PM

How to determine minimum Sample Size needed

For Proportion

$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$ with some algebra $n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$

if decimal Round-up

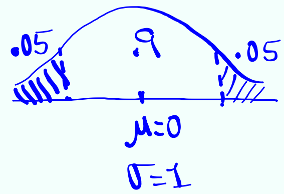
If $\hat{p}$ & $\hat{q}$ are both unknown, use .5 for each

$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2$

Jul 22-5:02 PM

Suppose $E = .04$, $\hat{p} = .7$

Find minimum Sample Size needed for 90% C-level.



$$n = \hat{p}\,\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

$$= (.7)(.3)\left(\frac{1.645}{.04}\right)^2$$

$Z_{\alpha/2} = \text{invNorm}(.95, 0, 1)$

$\boxed{1.645}$

$n = 355.166$

$n = 356$

If $\hat{p}$ & $\hat{q}$ were both unknown

$$n = .25\left(\frac{1.645}{.04}\right)^2$$

$= 422.816$

423

Jul 22-5:06 PM

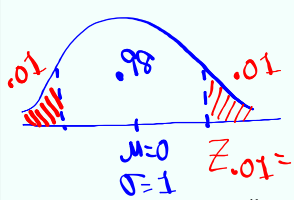
Find minimum Sample Size needed to Construct $\boxed{98\%\ \text{C-level}}$ for population Proportion with margin of error not to exceed 5% and

1) If $\hat{p} = .4$

$$n = \hat{p}\,\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

$$= (.4)(.6)\left(\frac{2.326}{.05}\right)^2$$

$= 519.386$ $\boxed{n = 520}$



$Z_{.01} = \text{invNorm}(.99, 0, 1) = 2.326$

2) If $\hat{p}$ & $\hat{q}$ are both unknown.

$$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{2.326}{.05}\right)^2 \approx$$

$541.0276 \rightarrow \boxed{542}$

Jul 22-5:13 PM

25 randomly selected 2B2B apts. had a mean monthly rent of \$ 2350.

$n=25$, $\bar{x}=2350$

It is known that standard deviation of monthly rent of all 2B2B Apts. is \$400. $\boxed{\sigma=400}$

C-level : .9

Find 90% Conf. interval for the mean rent of all 2B2B Apts.

$\boxed{\text{Z Interval } (\sigma \text{ known})}$ $<\mu<$

inpt: STATS

$\sigma=400$

$\bar{x}=2350$

$n=25$

C-level: .9

Calculate

T Interval (σ unknown)

$\boxed{2218<\mu<2482}$

$$E=\frac{2482-2218}{2}=\boxed{132}$$

$$\bar{x}=\frac{2482+2218}{2}=\boxed{2350}$$

Jul 22-5:34 PM

I randomly selected 18 servers. their mean hourly rate was \$23.50 with Standard deviation of \$4.75.

$n=18$

$\bar{x}=23.50$, $S=4.75$

NO C-level $\Rightarrow$ use .95

Find Conf. interval for the mean hourly rate of all servers.

Z Interval (σ known) $<\mu<$

$\boxed{\text{T Interval } (\sigma \text{ unknown})}$

inpt: Stats

$\boxed{\bar{x}=23.50}$

$S=4.75$

$n=18$

C-level: .95

Calculate

$df=n-1=17$

$\boxed{21.14<\mu<25.86}$

$$E=\frac{25.86-21.14}{2}=\boxed{2.36}$$

$$\bar{x}=\frac{25.86+21.14}{2}=\boxed{23.50}$$

Jul 22-5:44 PM

12 students were randomly selected. Here are their ages:

25	30	20	28
18	32	40	24
19	42	35	38

Find

1) $\bar{x} = 29.3$

2) $S = 8.3$

Round to 1-Dec.

use the rounded answer to find 99% Conf. interval for the mean age of all students.

Z Interval (σ Known)

$\boxed{\text{T Interval } (\sigma \text{ unknown})}$

$\boxed{21.9 < \mu < 36.7}$

Inpt: Stats

$\bar{x} = 29.3$

$S = 8.3$

$n = 12$

C-level: .99

Calculate

Jul 22-5:53 PM

Minimum Sample Size needed for pop. Mean:

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

with some Algebra

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

if decimal $\Rightarrow$ Always Round-up

If σ is unknown, use S

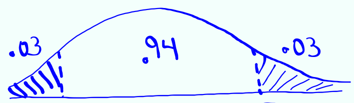
$$n = \left(\frac{Z_{\alpha/2} \cdot S}{E} \right)^2$$

Jul 22-6:02 PM

Find minimum Sample Size needed to Construct 94% Conf. interval for population mean if $\sigma = 12$ and $E = 5$.

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left(\frac{1.881 \cdot 12}{5}\right)^2 = 20.37\ldots$$

$n = 21$



$Z_{.03} = \text{invNorm}(.97, 0, 1) = 1.881$

Redo with $E = 4$ $\Rightarrow n = \left(\frac{1.881 \cdot 12}{4}\right)^2 \approx \boxed{32}$

Redo with 99% C-level and $E = 4$ $\Rightarrow n = \left(\frac{2.576 \cdot 12}{4}\right)^2 \approx \boxed{60}$

$Z = \text{invNorm}(.995, 0, 1) = \boxed{2.576}$

Jul 22-6:05 PM

I randomly Selected 20 exams, here are the Scores:

72	86	93	90	85	70	100
68	95	88	76	70	65	95
100	80	90	70	50	60	

Find $\bar{x}$ & S, Round to whole #

$\bar{x} = 80$, $S = 14$, $n = 20$

Find Conf. interval for the mean Score of all exams.

T Interval

NO C-level .95

inpt: Stats

$\bar{x} = 80$

$S = 14$

$n = 20$

C-level: .95

Calculate

$\boxed{73 < \mu < 87}$

$E = \frac{87 - 73}{2} = \boxed{7}$

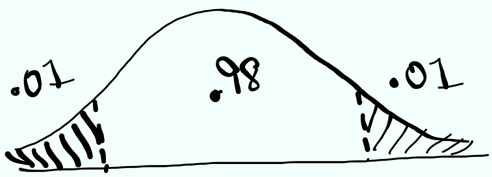
Jul 22-6:14 PM

Find min. Sample Size needed for # of exams if we wish to construct 98% C-level and margin of error not to exceed 5 pts.

use S

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left(\frac{2.326 \cdot 14}{5}\right)^2 \approx \boxed{43}$$

SG 22 & 23



$z_{.01} = \text{invNorm}(.99, 0, 1) = 2.326$

Jul 22-6:21 PM

Preview to SG 24

Testing claim

Claim could be about any parameter.

We need to test the claim to determine its validity.

If we conclude that	then
claim is valid	we fail-to-reject it
claim is invalid	we reject it.

Final Conclusion:

Reject the claim OR $\overbrace{\text{Fail-to-Reject the claim}}^{\text{Support}}$

Jul 22-6:40 PM

Possible error :

If claim is valid but we reject it.

If claim is invalid but we fail-to-reject it.

Testing Methods:

1) Traditional Method
2) P- Value Method

3) Confidence Interval Method

Regardless of the method used, final conclusion will be the same.

Reject the claim OR FTR the claim

Jul 22-6:45 PM

Testing Types:

1) Right-Tail Test RTT

2) Left - Tail Test LTT

3) Two - Tail Test TTT

with every testing, there is a value α, $0<\alpha<1$, α is called Significance level.

If α not given $\Rightarrow$ Use .05

Jul 22-6:50 PM

CNN claims that 40% of voters have voted by party line.

$p = .4$ claim

The college claims that the mean age of all college students is at most 32.5 yrs.

claim $\mu \leq 32.5$

I claim standard deviation of scores of all exams is more than 10.

$\sigma > 10$ claim

Jul 22-6:53 PM

Class QZ 7

Consider a binomial prob. dist with $n = 100$ and $p = .5$. x is # of successes

1) $P(x \leq 60)$ = binomcdf(100, .5, 60) = $\boxed{.982}$

2) $P(x = 45)$ = binompdf(100, .5, 45) = $\boxed{.048}$

Jul 22-6:58 PM